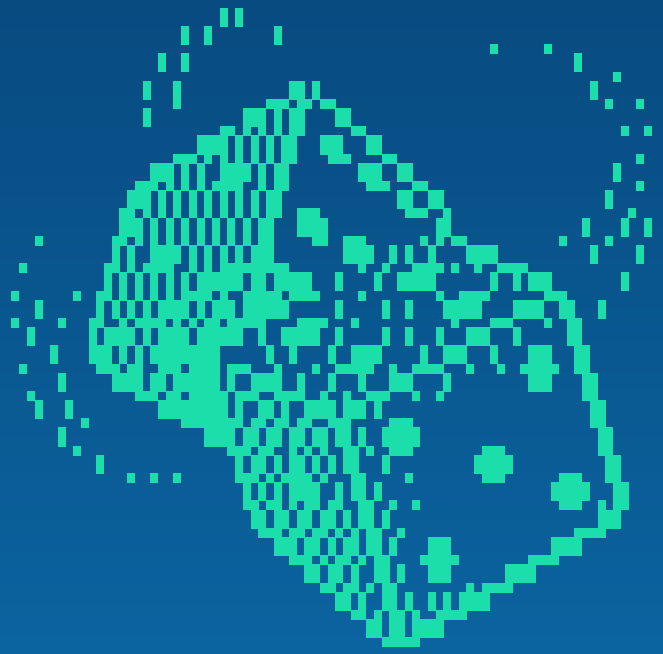
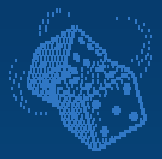
Entropy based Tests for Randomness and Applications to Cryptographic Generators

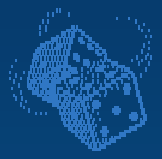


Stefan Wegenkittl
pLab-Group of Prof. Peter Hellekalek
Dept. of Mathematics
University of Salzburg
Stefan.Wegenkittl@sbg.ac.at
http://random.mat.sbg.ac.at



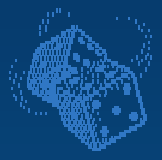
New Challenges for Empirical Testing

- (Pseudo)randomness: (Monte Carlo)
  typical correlations: repetitive structures



New Challenges for Empirical Testing

- (Pseudo)randomness: (Monte Carlo)
  typical correlations: repetitive structures
- (Pseudo)randomness: (Cryptography)
  typical correlations: much broader range of defects

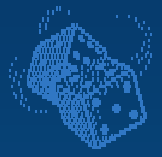


New Challenges for Empirical Testing

- (Pseudo)randomness: (Monte Carlo)
  typical correlations: repetitive structures
- (Pseudo)randomness: (Cryptography)
  typical correlations: much broader range of defects

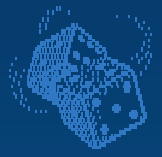
$\Rightarrow$ wanted:
universal tests that can detect
any systematic deviation from randomness

? cost of universality



Why Entropy

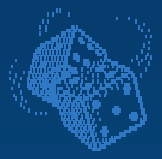
Entropy is



Why Entropy

Entropy is 

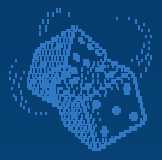
- well understood concept for measuring randomness



Why Entropy

Entropy is 

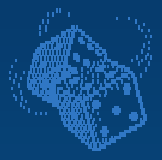
- well understood concept for measuring randomness
- applicable to finite sequences



Why Entropy

Entropy is 

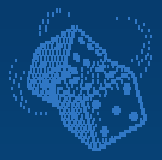
- well understood concept for measuring randomness
- applicable to finite sequences
- related to cryptographic security



Why Entropy

Entropy is 

- well understood concept for measuring randomness
- applicable to finite sequences
- related to cryptographic security
- related to established tests for randomness in Monte Carlo



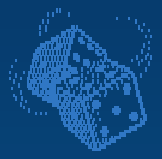
Interpretation

Interpretation: Per-Bit Entropy

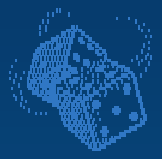
$$\frac{1}{\log_2 \text{ (size of state space)}} \cdot \text{ Entropy}$$

of a cryptographic source

is equal to factor by which effective keysize is reduced

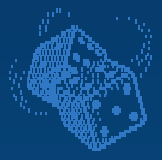


How to apply Entropy based tests



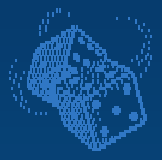
How to apply Entropy based tests

- different test principles (return times, frequencies)



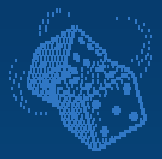
How to apply Entropy based tests

- different test principles (return times, frequencies)
- how to parameterize?



How to apply Entropy based tests

- different test principles (return times, frequencies)
- how to parameterize?
- what can we expect?

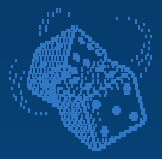


How to apply Entropy based tests

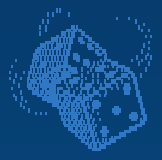
- different test principles (return times, frequencies)
- how to parameterize?
- what can we expect?
- correlations between tests?

Question:

Entropy based testing universal, i.e. no other tests are required?



Entropy Primer



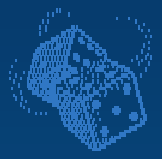
Entropy of Ergodic Prozesses

The scaled logarithm of the probability of a random path

of length n converges to a limit

$$-\frac{1}{n}\log\left(P[\text{ random Path }]\right) \xrightarrow{\text{a.s.}} H \quad \ldots \text{ Entropy}$$

(Shannon-McMillan-Breimann)

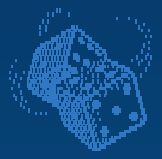


Order κ Markov Chains

The process $X = (X_i)_{i \geq 0}, \quad X_i \in \mathcal{A} := \{1, 2, \ldots, m\}$ is a

homogenous Markov-Chain of order κ

iff



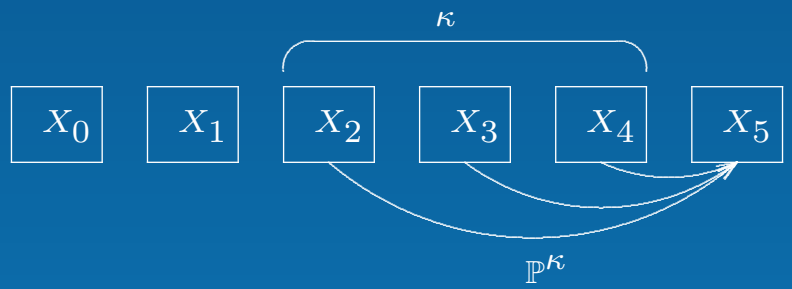
Order κ Markov Chains

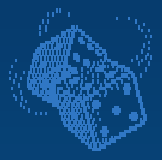
The process $X = (X_i)_{i \geq 0}, \quad X_i \in \mathcal{A} := \{1, 2, \ldots, m\}$ is a

homogenous Markov-Chain of order κ

iff

$$P[X_i = a_i | X_{i-1} = a_{i-1}, \ldots, X_0 = a_0] = p_{a_{i-\kappa}, \ldots, a_{i-1}, a_i}$$



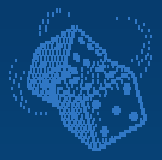


Ergodic Order κ Markov Chains

If transition probabilities

$$\mathbb{P}^{\kappa} = \left(p_{a_{i-\kappa},\ldots,a_{i-1},a_i}\right)_{\mathbf{a}\in\mathcal{A}^{(\kappa+1)}}$$

define an irreducible and aperiodic chain,



Ergodic Order κ Markov Chains

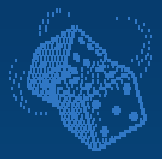
If transition probabilities

$$\mathbb{P}^{\kappa} = \left(p_{a_{i-\kappa},\ldots,a_{i-1},a_i}\right)_{\mathbf{a}\in\mathcal{A}^{(\kappa+1)}}$$

define an irreducible and aperiodic chain,

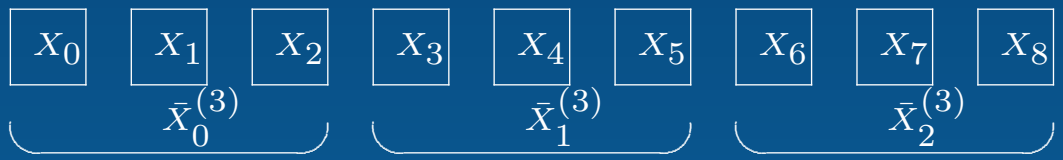
there exists a unique stable distribution

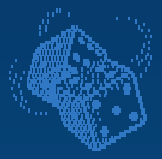
$$\left(\pi^{(\kappa)}_{a_1,\ldots,a_\kappa}\right)_{\mathbf{a}\in\mathcal{A}^{\kappa}} \text{ with } \pi^{(\kappa)} = \pi^{(\kappa)} \cdot \mathbb{P}^{\kappa}$$



Derived Processes of Order κ Chains

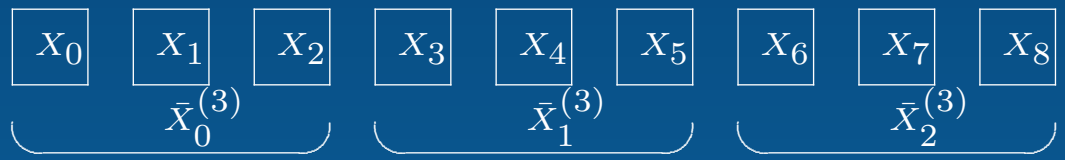
$$\bar{X}_i^{(r)} = (X_{r\cdot i}, \ldots, X_{r\cdot i+r-1}) \quad \ldots . \text{non-overlapping } r\text{-tuples}$$



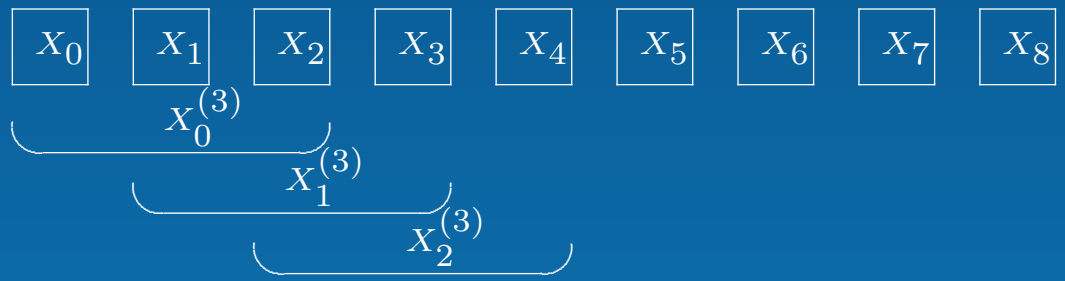


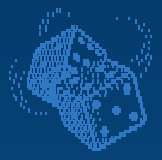
Derived Processes of Order κ Chains

$$\bar{X}_i^{(r)} = (X_{r\cdot i}, \ldots, X_{r\cdot i+r-1}) \quad \ldots . \text{non-overlapping } r\text{-tuples}$$



$$X_i^{(r)} = (X_i, \ldots, X_{i+r-1}) \quad \ldots\ldots\ldots\ldots \text{overlapping } r\text{-tuples}$$

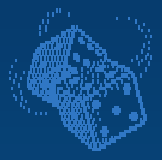




Derived Processes of Order κ Chains (ii)

If $r > \kappa$, the process of overlapping r tuples

$(X_i^{(r)})_{i \geq 0}$ becomes ordinary (order 1) chain



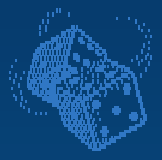
Derived Processes of Order κ Chains (ii)

If $r > \kappa$, the process of overlapping r tuples

$(X_i^{(r)})_{i \geq 0}$ becomes ordinary (order 1) chain

with stable distribution

$$\pi^{(r)}_{a_1,\ldots,a_r} = \pi^{(\kappa)}_{a_1,\ldots,a_\kappa} \cdot \prod_{l=\kappa+1}^{r} p_{a_{l-\kappa},a_{l-\kappa+1},\ldots,a_l}$$

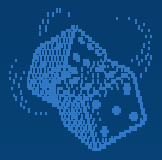


Entropy of Order κ Chains

For $r > \kappa$ Birkhoff's theorem gives

$$H = H(\mathcal{A}, \mathbb{P}^\kappa) = -\sum_{\mathbf{a}\in\mathcal{A}^r} \pi_{\mathbf{a}}^{(r)} \log_2 \underbrace{\left(\frac{\pi_{\mathbf{a}}^{(r)}}{\pi_{\mathbf{a}'}^{(r-1)}} \right)}_{\text{prob. of last step}}$$

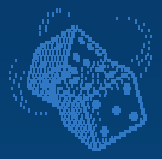
with $\mathbf{a}' = (a_1, \ldots, a_{r-1})$.



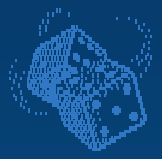
Entropy of Order 1 Chains

For ordinary chains, this gives:

$$H(\mathcal{A}, \mathbb{P}) = -\sum_{i \in \mathcal{A}} \pi_i \sum_{j \in \mathcal{A}} p_{ij} \log_2 p_{ij}$$



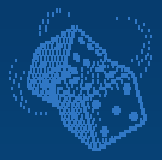
Computing Entropy of Chains



Using stable distributions: Chain Rule

For $r > \kappa$

$$H(\mathcal{A}, \mathbb{P}^\kappa) = H(\pi^{(r)}) - H(\pi^{(r-1)}),$$



Using stable distributions: Chain Rule

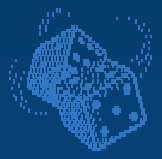
For $r > \kappa$

$$H(\mathcal{A}, \mathbb{P}^\kappa) = H(\pi^{(r)}) - H(\pi^{(r-1)}),$$

where

$$H(\pi^{(r)}) = -\sum_{\mathbf{a}\in\mathcal{A}^r} \pi^{(r)}_{\mathbf{a}} \log_2 \pi^{(r)}_{\mathbf{a}}$$

is the Entropy of the distribution $\pi^{(r)}$

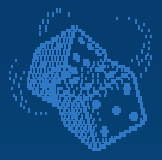


Using Return Times - Overlapping Case

Let

$$T^{(r)} = \min\left\{i \geq r : X_i^{(r)} = X_0^{(r)}\right\}$$

be the first (overlapping) return time of order r.



Using Return Times - Overlapping Case

Let

$$T^{(r)} = \min\left\{ i \geq r : X_i^{(r)} = X_0^{(r)} \right\}$$

be the first (overlapping) return time of order r.

Wyner & Ziv:

$$\frac{\log_2\left(T^{(r)}\right)}{r} \xrightarrow{\mathrm{p}} H, \text{ as } r \longrightarrow \infty$$

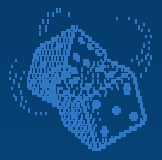
Pointwise Theorem: Ornstein, Weiss

Using Return Times - Non-Overlapping Case

Let

$$\bar{T}^{(r)} = \min\left\{ i \geq 1 : \bar{X}_i^{(r)} = \bar{X}_0^{(r)} \right\}$$

be the first (non-overlapping) return time of order r.



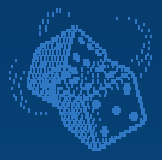
Expected Non-Overlapping Return Times

If $r \geq \kappa$,

$$E\left[\bar{T}^{(r)} | \bar{X}_0^{(r)} = \mathbf{a}\right] = \frac{1}{\pi_{\mathbf{a}}^{(r)}}, \quad \mathbf{a} \in \mathcal{A}^r$$

and, consequently,

$$E\left[\bar{T}^{(r)}\right] = m^r$$



Expected Non-Overlapping Return Times

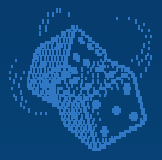
If $r \geq \kappa$,

$$E\left[\bar{T}^{(r)} | \bar{X}_0^{(r)} = \mathbf{a}\right] = \frac{1}{\pi_{\mathbf{a}}^{(r)}}, \quad \mathbf{a} \in \mathcal{A}^r$$

and, consequently,

$$E\left[\bar{T}^{(r)}\right] = m^r$$

$E[\cdot]$ contains no information on chain!

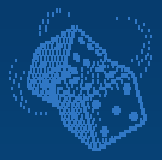


Expected Logarithm of Return Times

Thus: use $E[\log_2(\cdot)]$

Maurer, similar to Willems:

$$\frac{E\left[\log_2\left(\bar{T}^{(r)}\right)\right]}{r} \longrightarrow H$$

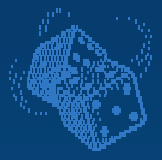


Expected Logarithm of Return Times (ii)

Speed of convergence: Let

$$E(r) = (r-1) \cdot H(\mathcal{A}, \mathbb{P}^{\kappa}) + H(\pi^{(r)}),$$

then



Expected Logarithm of Return Times (ii)

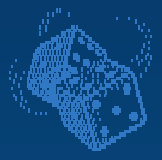
Speed of convergence: Let

$$E(r) = (r-1) \cdot H(\mathcal{A}, \mathbb{P}^\kappa) + H(\pi^{(r)}),$$

then

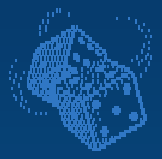
$$\lim_{r \to \infty} \left(E\left[\log_2 \left(\bar{T}^{(r)} \right) \right] - E(r) \right) = \frac{\gamma}{\ln(2)}$$

Wegenkittl, IEEE 2002



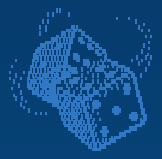
Summary

- Entropy well defined for order κ chains
- Compute by either



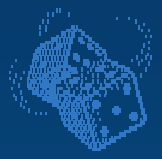
Summary

- Entropy well defined for order κ chains
- Compute by either
  - stable distribution of overlapping tuples and chain rule

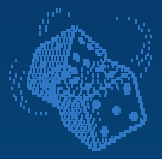


Summary

- Entropy well defined for order κ chains
- Compute by either
  - stable distribution of overlapping tuples and chain rule
  - expected logarithm of return time of non-overlapping tuples



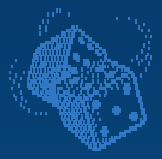
Tests for Randomness based on Entropy



Tests for Randomness based on Entropy

H_0: X is i.i.d. uniform on $\mathcal{A}$

H_1: X is not H_0 but stationary, ergodic chain



Tests for Randomness based on Entropy

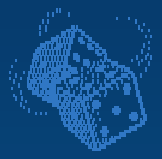
H_0: X is i.i.d. uniform on $\mathcal{A}$

H_1: X is not H_0 but stationary, ergodic chain

Idea: given realization

$$x = (x_0, x_1, \ldots, x_{n-1})$$

estimate Entropy H (or related property) of X



Tests for Randomness based on Entropy

H_0: X is i.i.d. uniform on $\mathcal{A}$

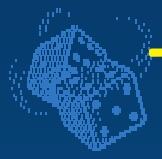
H_1: X is not H_0 but stationary, ergodic chain

Idea: given realization

$$x = (x_0, x_1, \ldots, x_{n-1})$$

estimate Entropy H (or related property) of X

Reject H_0 if result unlikely under H_0.

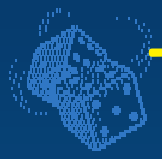


Tests for Randomness based on Entropy (ii)

Universality:

H_0: $H = \log_2(m)$

H_1: $H < \log_2(m)$



Tests for Randomness based on Entropy (ii)

Universality:

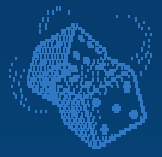
H_0: $H = \log_2(m)$

H_1: $H < \log_2(m)$

Estimator of H will be able to

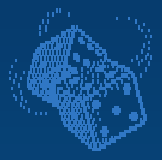
detect any possible deviation from H_0

provided n is large enough.



Construction of Entropy Tests

Goal: estimate Entropy

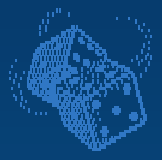


Tests based on relative frequency

Let

$$\hat{\pi}_{\mathbf{a}}^{(r)} = \frac{1}{n} \# \left\{ 0 \leq i < n \ : \ \bar{X}_i^{(r)} = \mathbf{a} \right\}$$

then $\hat{\pi}_{\mathbf{a}}^{(r)}$ is unbiased consistent estimator for $\pi_{\mathbf{a}}^{(r)}$



Tests based on relative frequency

Let

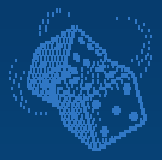
$$\hat{\pi}_{\mathbf{a}}^{(r)} = \frac{1}{n} \# \left\{ 0 \le i < n \ : \ \bar{X}_i^{(r)} = \mathbf{a} \right\}$$

then $\hat{\pi}_{\mathbf{a}}^{(r)}$ is unbiased consistent estimator for $\pi_{\mathbf{a}}^{(r)}$

Apply Chain-Rule: For $r > \kappa$,

$$\hat{H}_f := H(\hat{\pi}^{(r)}) - H(\hat{\pi}^{(r-1)})$$

is asymptotically unbiased consistent estimator for H.



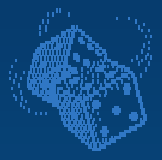
Tests based on relative frequency (ii)

Normalization:

dense : $2n(\log_2(m) - \hat{H}_f) \xrightarrow{\mathrm{d}} \chi^2_{m^r - m^{r-1}}$, as $n \to \infty$

sparse : $\frac{\hat{H}_f - \mu}{\sigma} \xrightarrow{\mathrm{d}} N(0,1)$ as $n, r \to \infty, \frac{m^r}{n} \to \lambda$

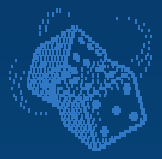
r . . . dimension of test m . . . granularity



Tests based on relative frequency (iii)

Examples for H_f:

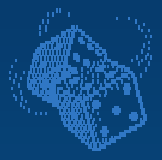
Approx. Entropy	Pincus et. al., NIST battery: discrete case, which is equivalent to $\hat{H}_f$
Modified ApEn	Ruhkin, asymptotically and practically equivalent to $\hat{H}_f$
Overlapping I-Divergence	Wegenkittl: connection to serial testing



Tests based on relative frequency (iv)

H_f belongs to the class of Power-Divergence Tests :

- statistical goodness-of-fit tests
- various likelyhood functions
- statistically equivalent in dense case
- very efficient in sparse case (L'Ecuyer, Simard, Wegenkittl)



Tests based on relative frequency (v)

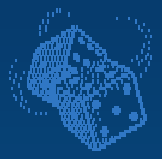
Standard overlapping serial test (Pearson, Good): let

$$\psi^{(r)} := n \sum_{\mathbf{a} \in \mathcal{A}^r} \frac{\left(\hat{\pi}_{\mathbf{a}}^{(r)} - \pi_{\mathbf{a}}^{(r)}\right)^2}{\pi_{\mathbf{a}}^{(r)}}$$

and put

$$\hat{\chi} = \psi^{(r)} - \psi^{(r-1)} \xrightarrow{\mathrm{d}} \chi^2_{m^r - m^{r-1}}$$

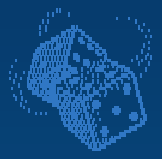
Examples: Monkey Tests in Diehard, Serial test in NIST



Tests based on relative frequency (vi)

Universality: If X is an order κ Markov chain

and provided $r > \kappa$,

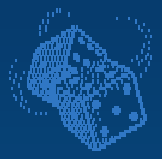


Tests based on relative frequency (vi)

Universality: If X is an order κ Markov chain

and provided $r > \kappa$,

$\hat{H}_f$ is able to detect any deviation from H_0



Tests based on relative frequency (vii)

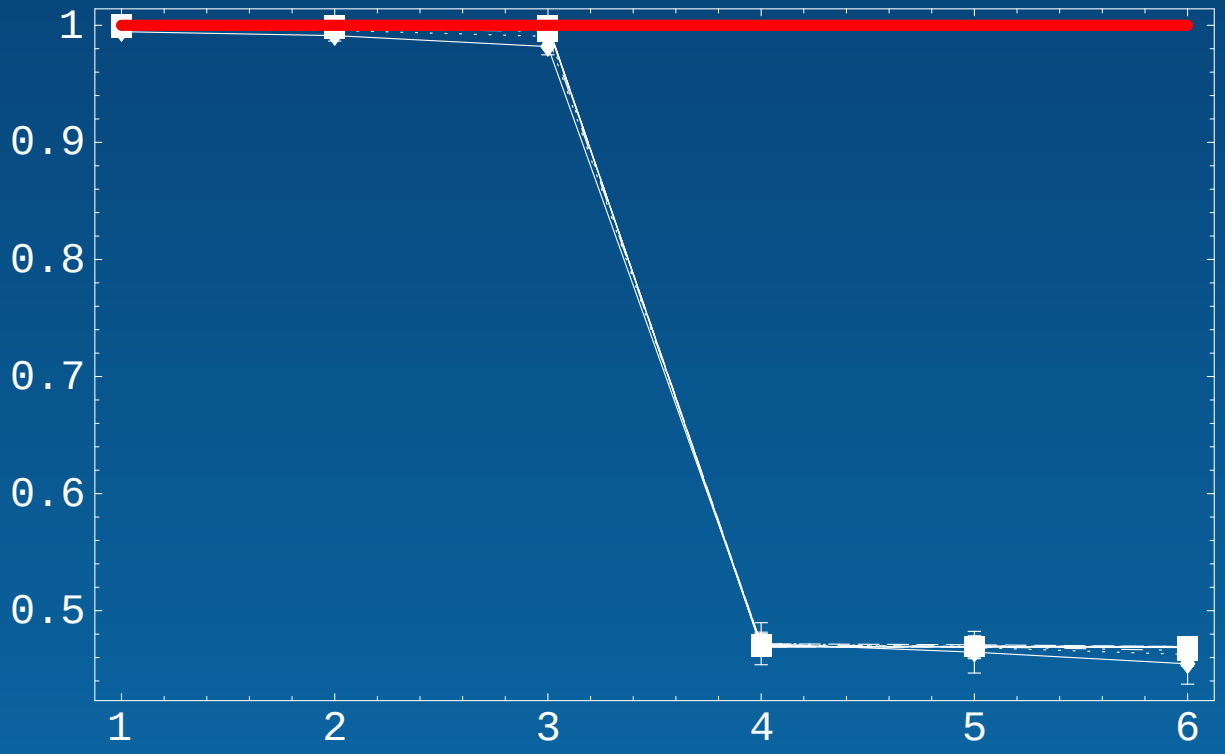
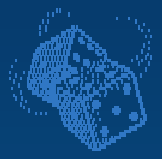


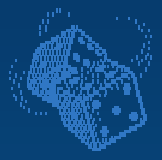
Fig. 1: Samples of $\hat{H}_f$ for an order-3 chain with $H = 0.469$



Tests based on return times

Maurer: estimate $E[\log_2(\bar{T}^{(r)})]$ by

$$\hat{H}_r := \frac{1}{r \cdot n} \sum_{i=Q+1}^{Q+n} \log_2 T(i),$$



Tests based on return times

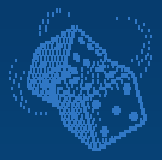
Maurer: estimate $E[\log_2(\bar{T}^{(r)})]$ by

$$\hat{H}_r := \frac{1}{r \cdot n} \sum_{i=Q+1}^{Q+n} \log_2 T(i),$$

with (letting $\bar{X}_{-1}^{(r)} := \bar{X}_i^{(r)}$)

$$T(i) = \min \left\{ 1 \le j \le i+1 \ : \ \bar{X}_{i-j}^{(r)} = \bar{X}_i^{(r)} \right\}$$

Q . . . warm-up, initialization



Tests based on return times (ii)

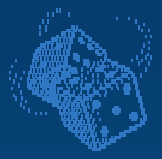
Normalization: compute (tables available)

$E[\hat{H}_r]$ (rather easy)

$V[\hat{H}_r]$ (rather complicated, only approx. available)

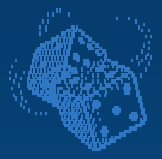
under H_0, then

$$\frac{\hat{H}_r - E[\cdot]}{\sqrt{V[\cdot]}} \xrightarrow{\mathrm{d}} N[0,1]$$



Tests based on return times (iii)

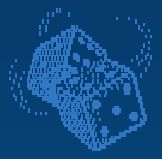
Universality: Maurer: $E\left[\log_2\left(\bar{T}^{(r)}\right)\right]$ detects any significant deviation from truly random bit source



Tests based on return times (iii)

Universality: Maurer: $E\left[\log_2\left(\bar{T}^{(r)}\right)\right]$ detects any significant deviation from truly random bit source

! case $r \longrightarrow \infty$ is clear



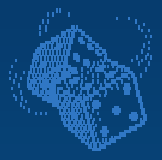
Tests based on return times (iii)

Universality: Maurer: $E\left[\log_2\left(\bar{T}^{(r)}\right)\right]$ detects any significant deviation from truly random bit source

! case $r \longrightarrow \infty$ is clear

? also true for $r \in \mathbb{N}$ fixed

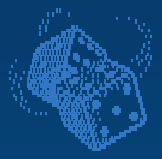
For any fixed $r \in \mathbb{N}$, we do not estimate H but only related property. Is this property sensitive to H_1?



Tests based on return times (vii)

Some precomputed values of $E[\log_2(.)]$ for $m = 2$

r	$E[r \cdot \hat{H}_r]$	$E[\hat{H}_r] - \frac{\gamma}{r \cdot \ln(2)}$
1	0.73265	1.5654
2	1.53744	1.1851
3	2.40161	1.0781
4	3.31122	1.0360
5	4.25343	1.0172
6	5.21771	1.0084



Tests based on return times (viii)

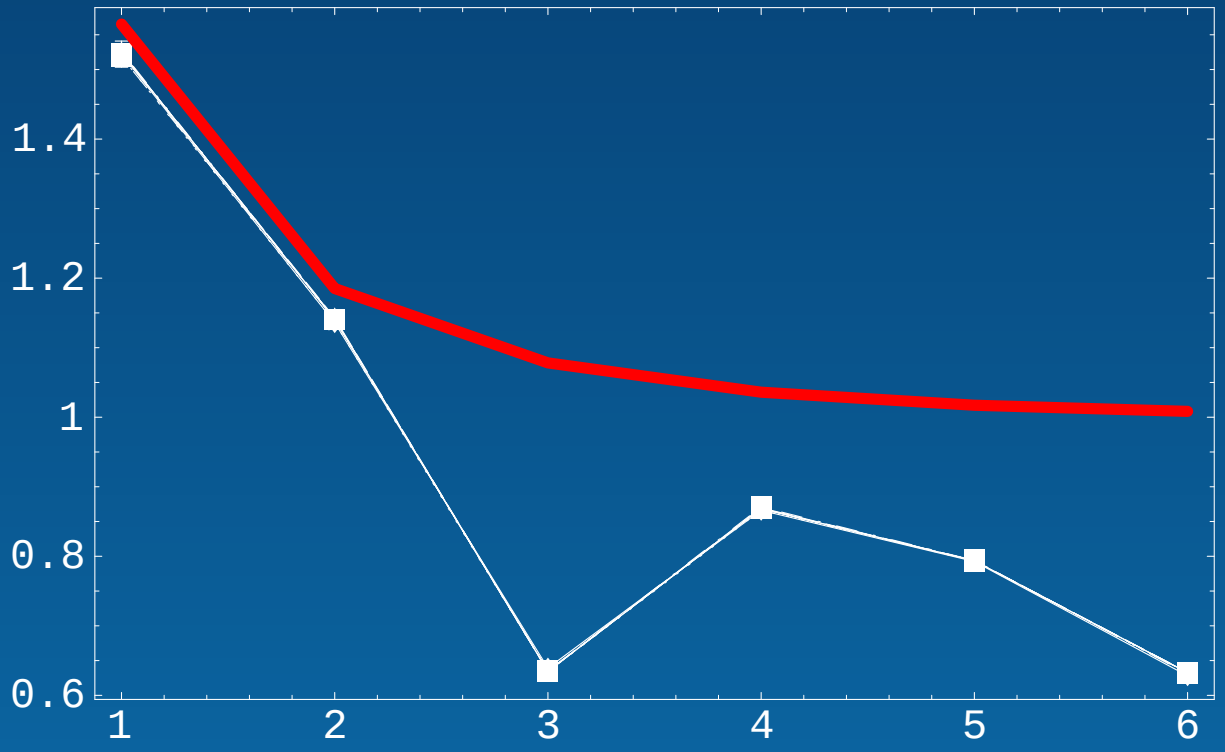
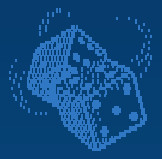


Fig. 2: Samples of $\hat{H}_r$ for an order-3 chain with $H = 0.469$



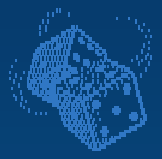
Summary

3 Types of Tests:

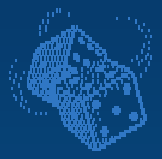
- Return Time based: Universal Test
- Relative Frequency based:

  a) Approximate Entropy, I-Divergence
  b) Serial Test, Monkey Test

Wegenkittl (IEEE 2002, PhD. Thesis): a) $\approx$ b)



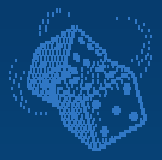
Return Time vs. Frequency: Sample Study



Return Time vs. Frequency: Sample Study

Idea: Simulate order κ chain with non-maximal Entropy

Vary "memory" κ of chain and "dimension" r of test

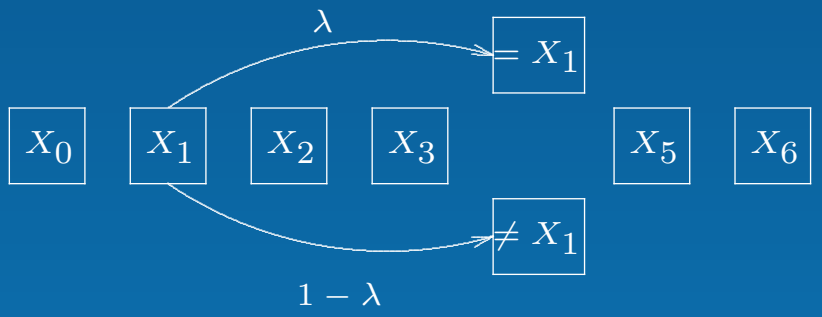


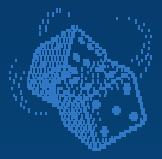
Sample Study: Defective Source

Let $m = 2$ (binary source) and

$$\tilde{\mathbb{P}} = (\tilde{p}_{ij})_{ij \in \mathcal{A}^2} = \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}$$

and define transition probabilities $p_{a_{i-\kappa},\ldots,a_{i-1},a_i} = \tilde{p}_{a_{i-\kappa}a_i}$

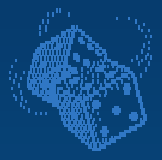




Defective Source Analysis

Then, for the chain $(\mathcal{A}, \mathbb{P}^\kappa(\lambda))$

- equidistribution on $\mathcal{A}^\kappa$ is the stable distribution
- $H(\text{stable distribution}) = 1$, consequently
- $H(\mathcal{A}, \mathbb{P}^\kappa) = H(\lambda, 1-\lambda) < 1 \Leftrightarrow \lambda \neq \frac{1}{2}$



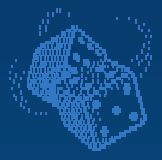
Defective Source Analysis

Then, for the chain $(\mathcal{A}, \mathbb{P}^\kappa(\lambda))$

- equidistribution on $\mathcal{A}^\kappa$ is the stable distribution
- $H(\text{stable distribution}) = 1$, consequently
- $H(\mathcal{A}, \mathbb{P}^\kappa) = H(\lambda, 1-\lambda) < 1 \Leftrightarrow \lambda \neq \frac{1}{2}$

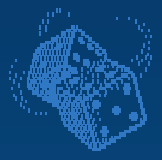
Some values of $H(\mathcal{A}, \mathbb{P}^\kappa)$:

λ	0.5	0.49	0.4	0.25	0.1
$H(\mathcal{A}, \mathbb{P}^\kappa)$	1.0	0.999711	0.970951	0.811278	0.468996



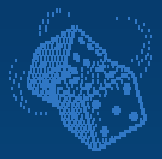
Sensitivity of $\hat{H}_f$ and $\hat{\chi}$

- $\pi^{(r)}$ is equidistribution for all $r \leq \kappa$
- so that $\hat{H}_f$ and $\hat{\chi}$ will not be able to see any defect until $r > \kappa$
- but (Chain Rule) $\hat{H}_f$ (and $\hat{\chi}$) will see defect for all $r > \kappa$

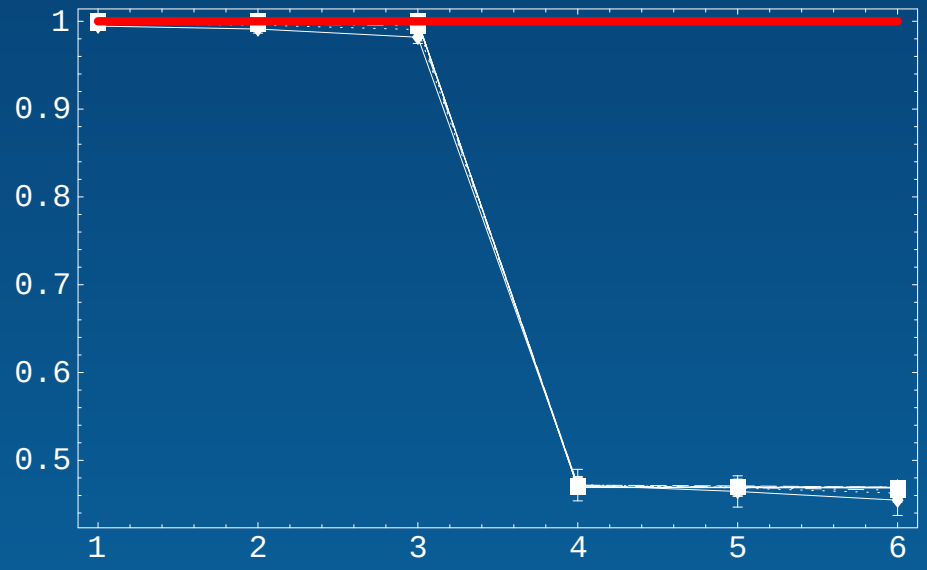


Sensitivity of $\hat{H}_r$

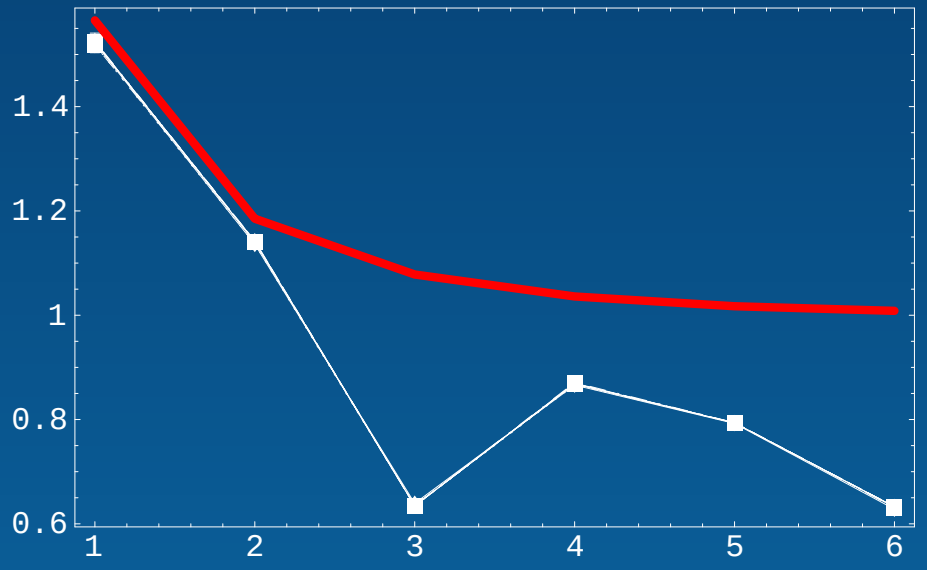
- $\hat{H}_r$ should be sensible to $\lambda \neq 1/2$ for all $r \geq 1$.
- is there any dependency on r with respect to fixed κ?



Example revisited

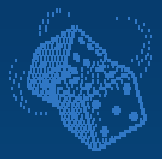


$\hat{H}_f$



$\hat{H}_r$

Fig. 3: Samples of $\hat{H}_f$ and $\hat{H}_r$ for $(\mathcal{A}, \mathbb{P}^3(1/2 - 0.4))$ with $H = 0.469$

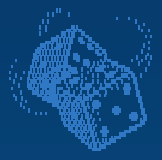


Speaking of samplesizes

In our setup,

ApEn and Serial Test use $\approx n$ samples,

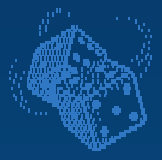
Universal Test uses $\approx n \cdot r$ samples plus warm-up



Speaking of samplesizes (ii)

$\log_2(n)$ needed to be able to detect Entropy defect (at 1% level of significance using $\hat{\chi}$):

r	1	2	4	8	12	16
$H = 0.999711$	15	15	16	19	23	27
$H = 0.468995$	4	4	5	8	12	16



Test Setup and Visualization

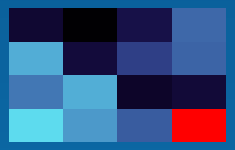
Compute 2-sided p-values according to given asymptotics

$\hat{H}_f$: dense case with χ^2 distribution

$\hat{\chi}$: dense case with χ^2 distribution

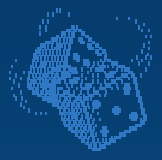
$\hat{H}_r$: normal distribution

Repeat each test 16 times and arrange results into small rectangle:

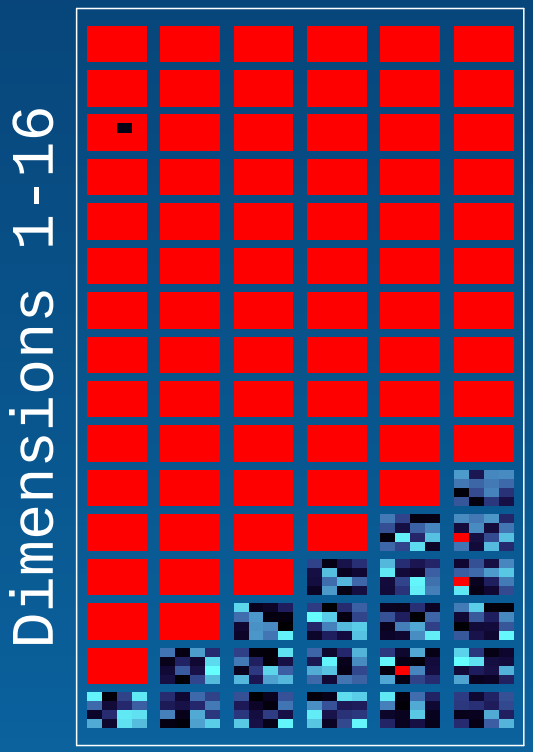


scale:

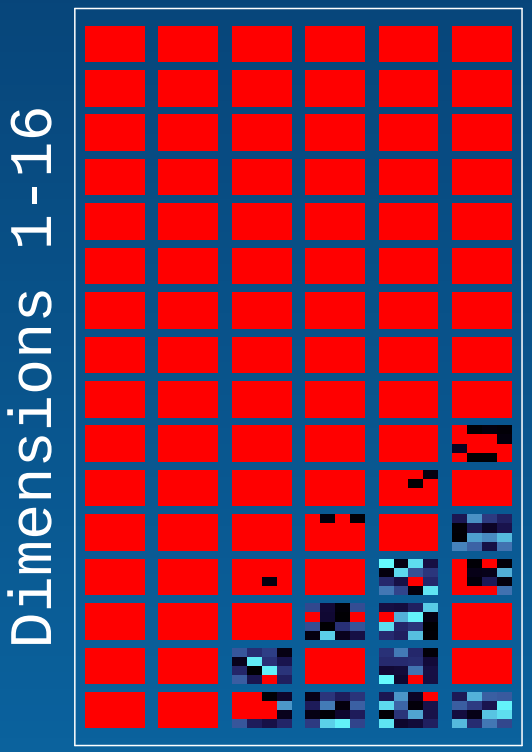
Red color indicates significance at 1% level.



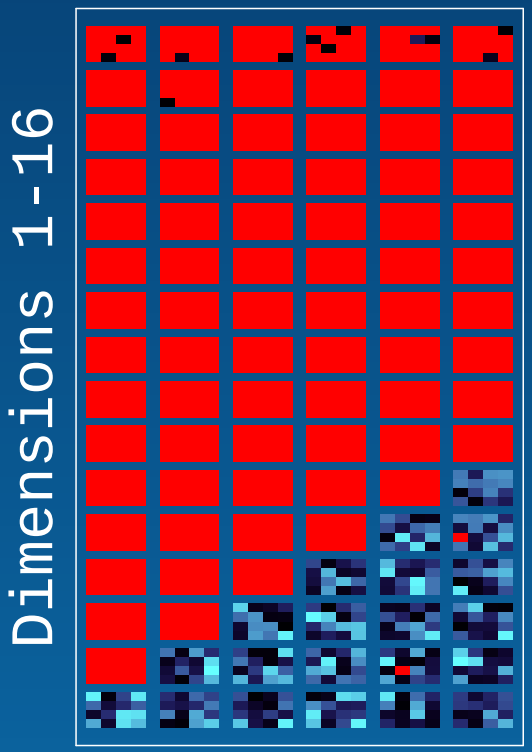
Easy case: $\lambda = 0.4$, $H = 0.970951$



$\hat{H}_f$, ApEn, I-Divergence

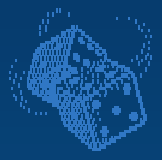


$\hat{H}_r$, Universal-Test

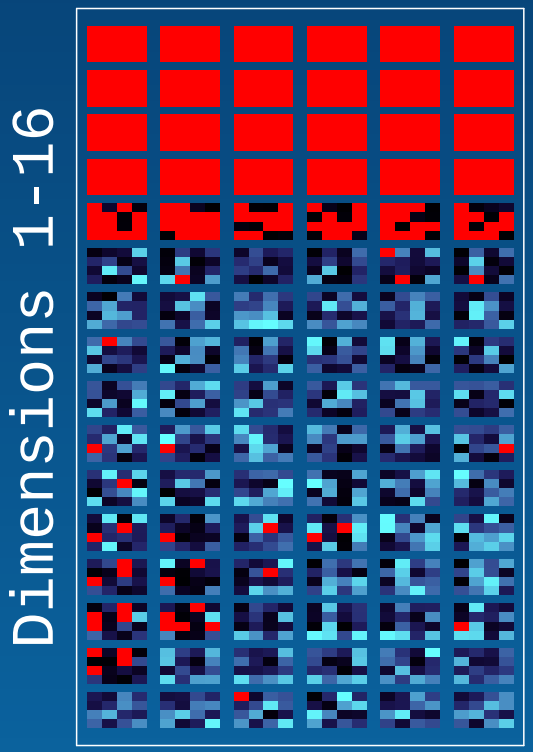


$\hat{\chi}$, Monkey-, Serial-Test

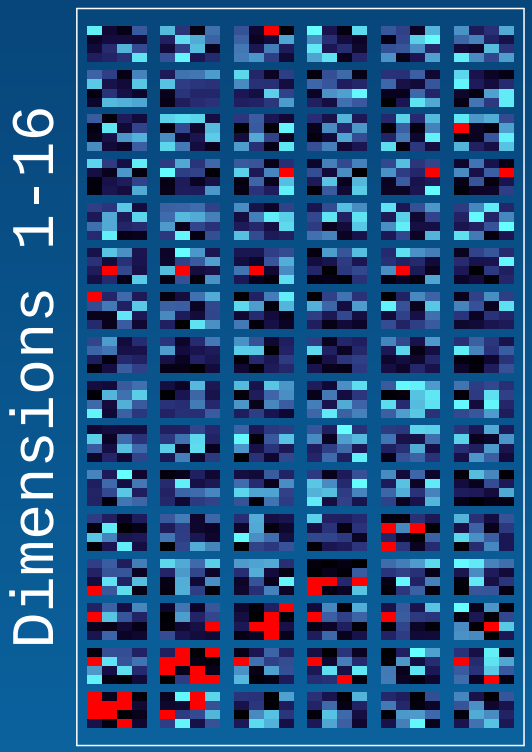
Parameters: $n = 2^{14}$, $1 \leq \kappa \leq 6$, $1 \leq r \leq 16$



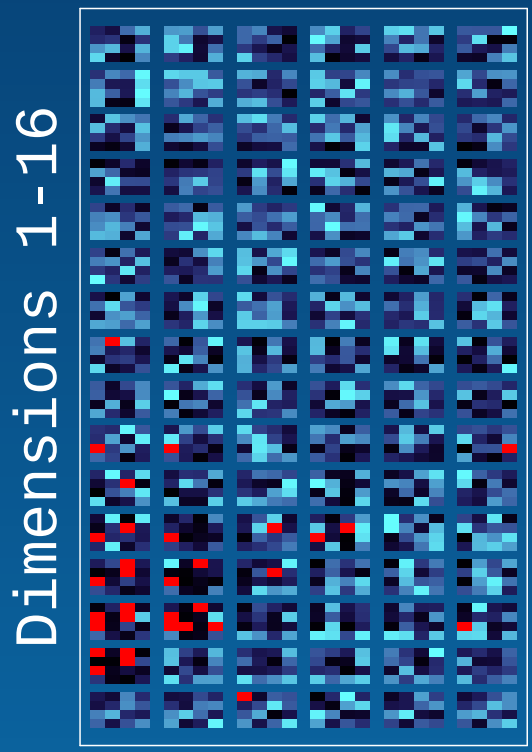
Hard case: $\lambda = 0.5$, $H = 1$, n too small



$\hat{H}_f$, ApEn, I-Divergence



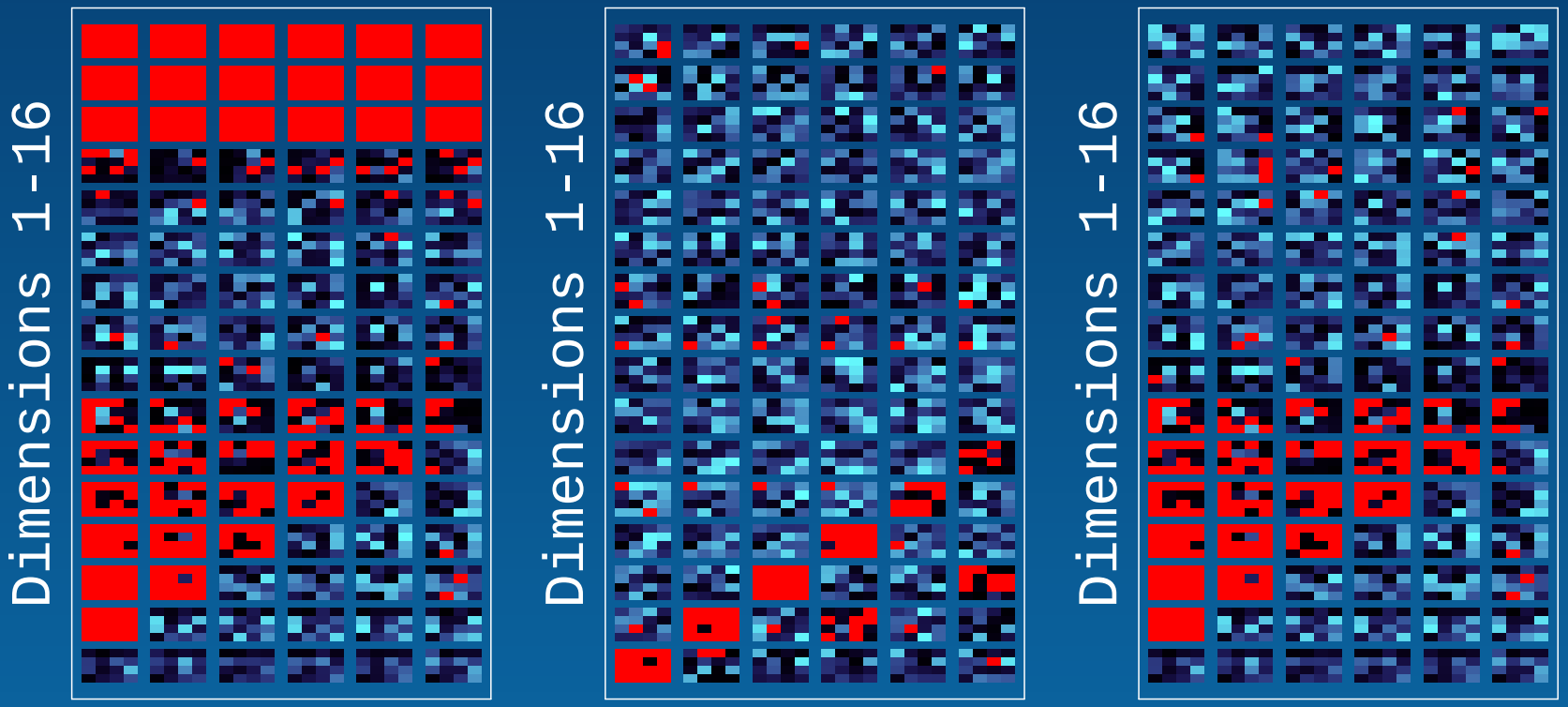
$\hat{H}_r$, Universal-Test



$\hat{\chi}$, Monkey-, Serial-Test

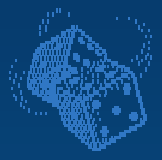
Parameters: $n = 2^{14}$, $1 \leq \kappa \leq 6$, $1 \leq r \leq 16$

Hard case: $\lambda = 0.49$, $H = 0.999711$, n still small

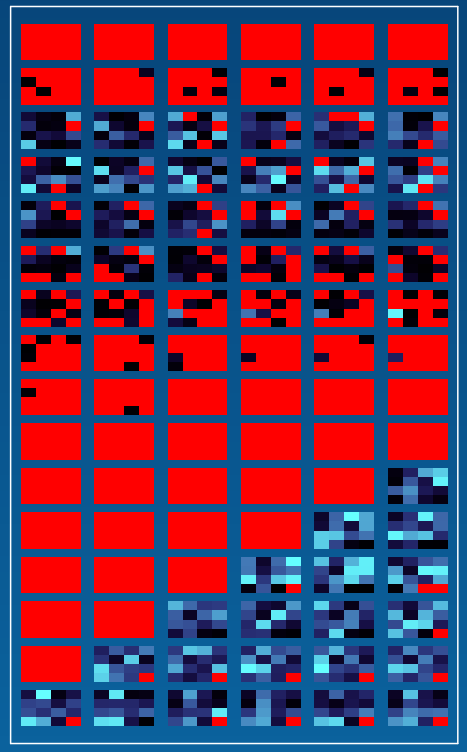


$\hat{H}_f$, ApEn, I-Divergence $\hat{H}_r$, Universal-Test $\hat{\chi}$, Monkey-, Serial-Test

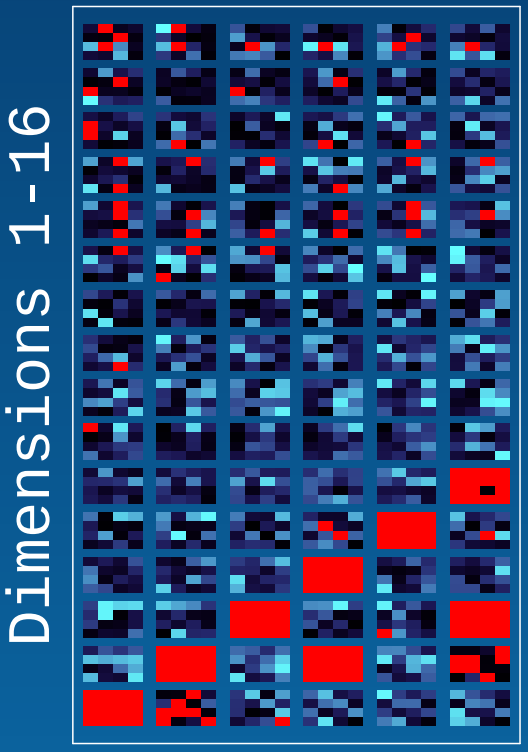
Parameters: $n = 2^{16}$, $1 \leq \kappa \leq 6$, $1 \leq r \leq 16$



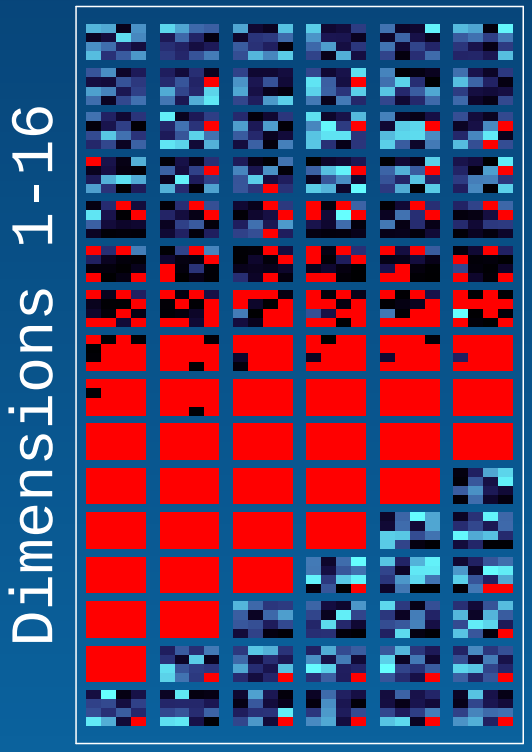
Hard case: $\lambda = 0.49$, $H = 0.999711$, n o.k.



$\hat{H}_f$, ApEn, I-Divergence

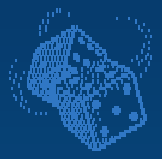


$\hat{H}_r$, Universal-Test



$\hat{\chi}$, Monkey-, Serial-Test

Parameters: $n = 2^{18}$, $1 \leq \kappa \leq 6$, $1 \leq r \leq 16$



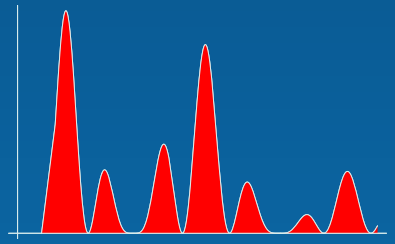
Summary: Degrees of Universality

special purpose — multi purpose — universal

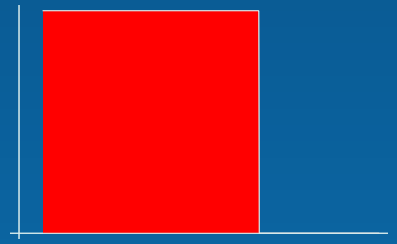
e.g. POKER TEST

UNIVERSAL TEST
vary DIMENSION

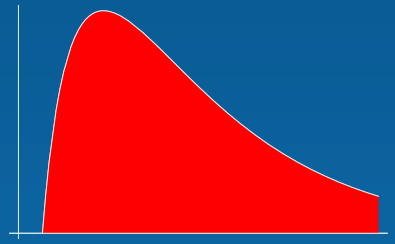
SERIAL TEST
vary DIMENSION, RESOLUTION



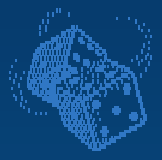
irregular



$r > \kappa$



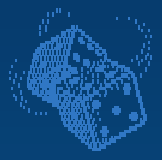
$r \in \mathbb{R}$



Summary: Correlation between Tests

Ability to detect H_1:

- ApEn and Serial Tests: similar, good results if r large enough
- Universal Test: good results if r "correct" (critical!)



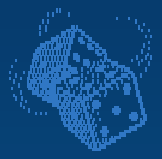
Summary: Correlation between Tests

Ability to detect H_1:

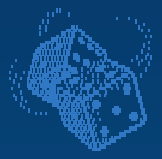
- ApEn and Serial Tests: similar, good results if r large enough
- Universal Test: good results if r "correct" (critical!)

Handling:

- Serial Tests: well understood, fast convergence
- ApEn is easy to interpret



Serial Tests in Applications

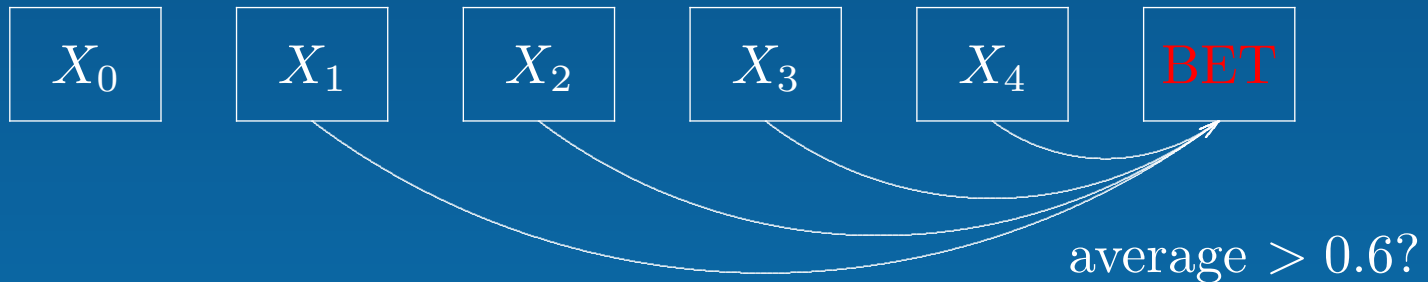


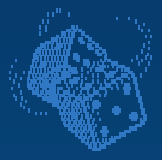
Dimension Reduction: Gambling Tests

Need $r \in 1 \ldots 256$ (generating keys for cryptosystems)

Technique of Gambling Tests (Wegenkittl 2000):

reduce test dimension, preserve typical RNG correlations



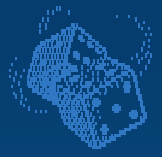


Application: RNG241 of PDH

Study: Hardware Generators in Serial Tests

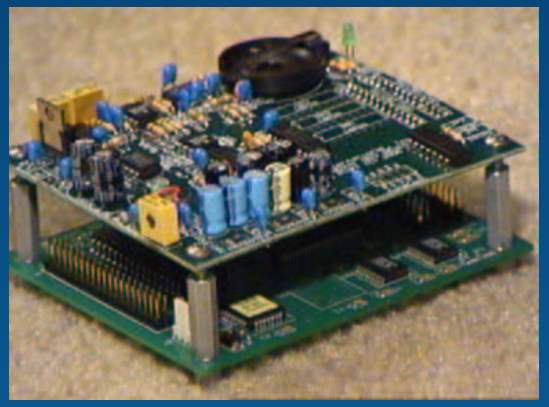
Example: RNG241

PDH International, Inc.
Precision Digital Hardware
1250 E. Hallandale Beach Blvd. PH2
Hallandale FL 33009, USA
www.pdhint.com
E-mail: contact@pdhint.com

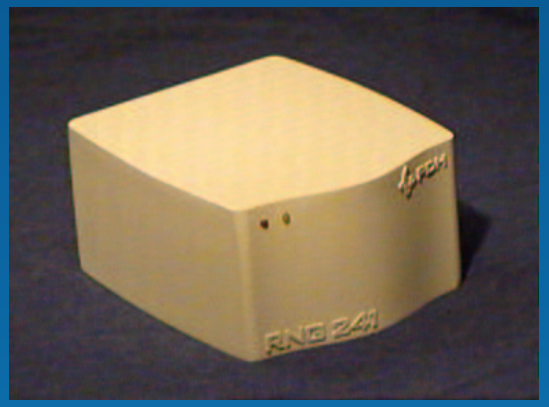


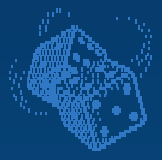
Application: RNG241 of PDH (ii)

radioactive α-decay RNG hardware
1 Kbps naturally perfect entropy
unaffected by external noise



plug & play serial-port box
integrated software solutions





Application: RNG241 of PDH (iii)

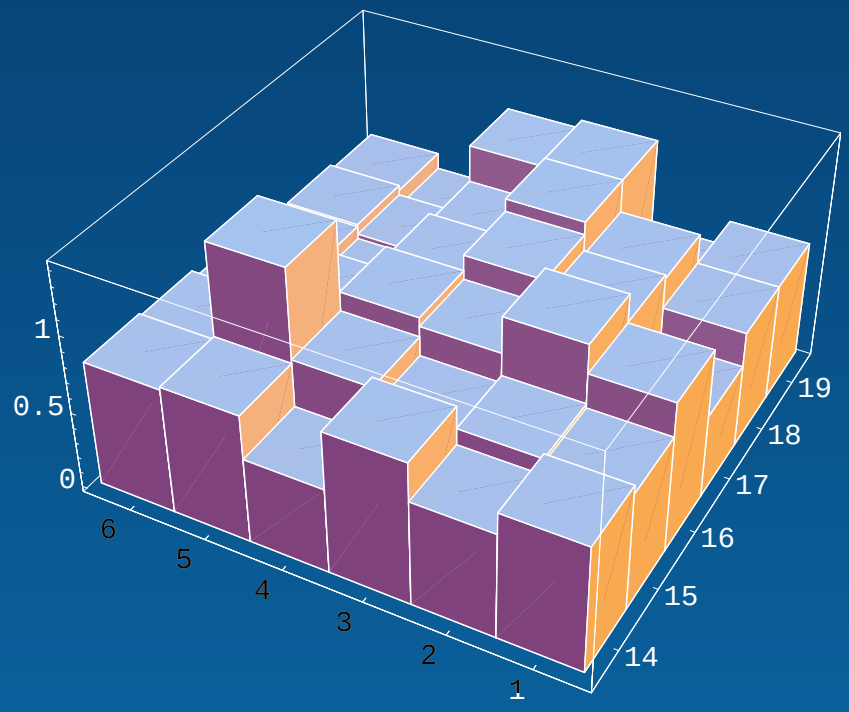
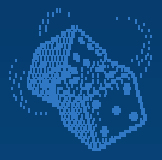


Fig. 4: Load Test: Each bar denotes the value of a single Kolmogorov-Smirnov statistic on 32 repetitions of the Load Test. Axes: dimension $s \in \{1, \ldots, 6\}$ and the dual logarithm of the level-one sample size ranging from 14 to 19.



Application: RNG241 of PDH (iv)

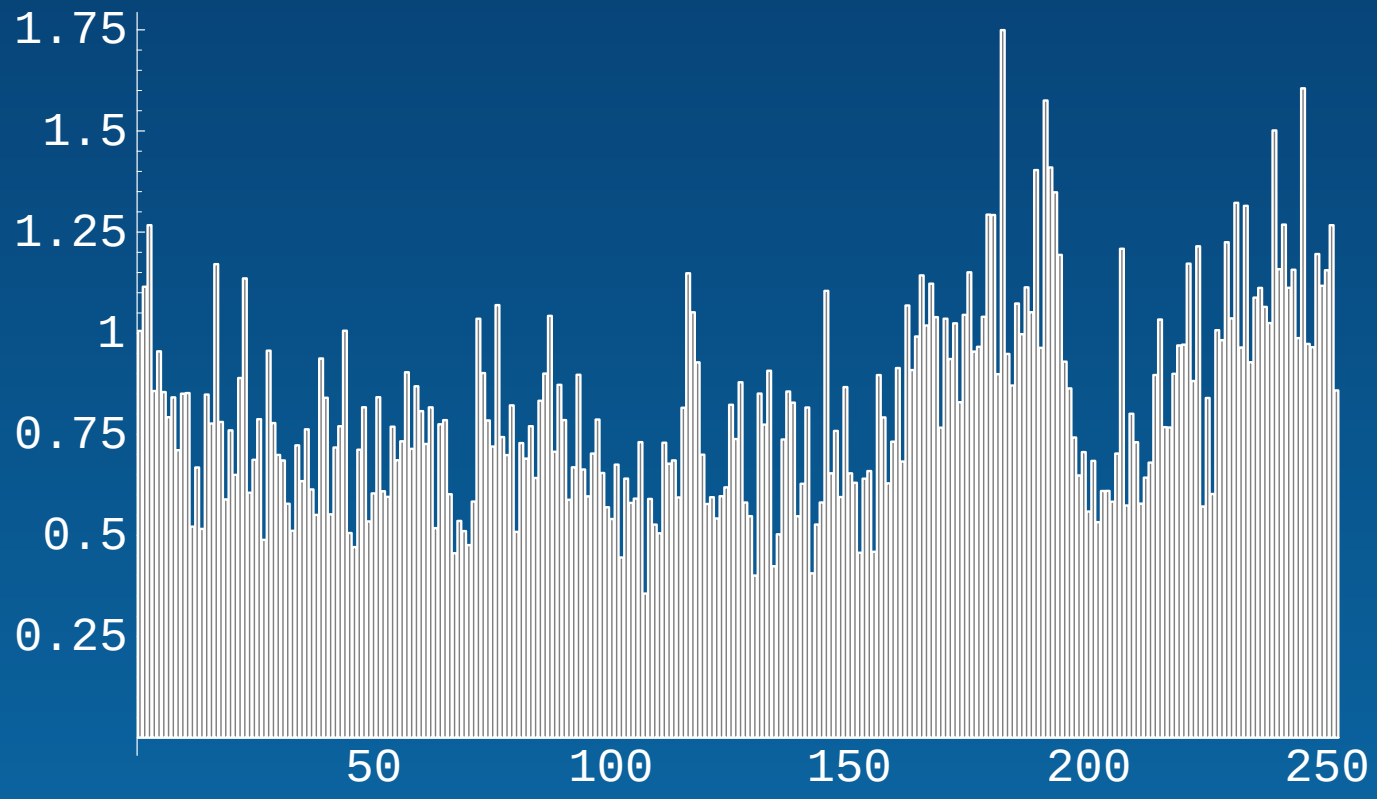
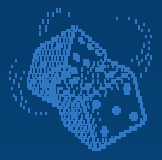


Fig. 5: A plot of the values of the Kolmogorov-Smirnov statistic for the Gambling Test with memory sizes $\lambda \in \{5, \ldots, 256\}$



Application: RNG241 of PDH (v)

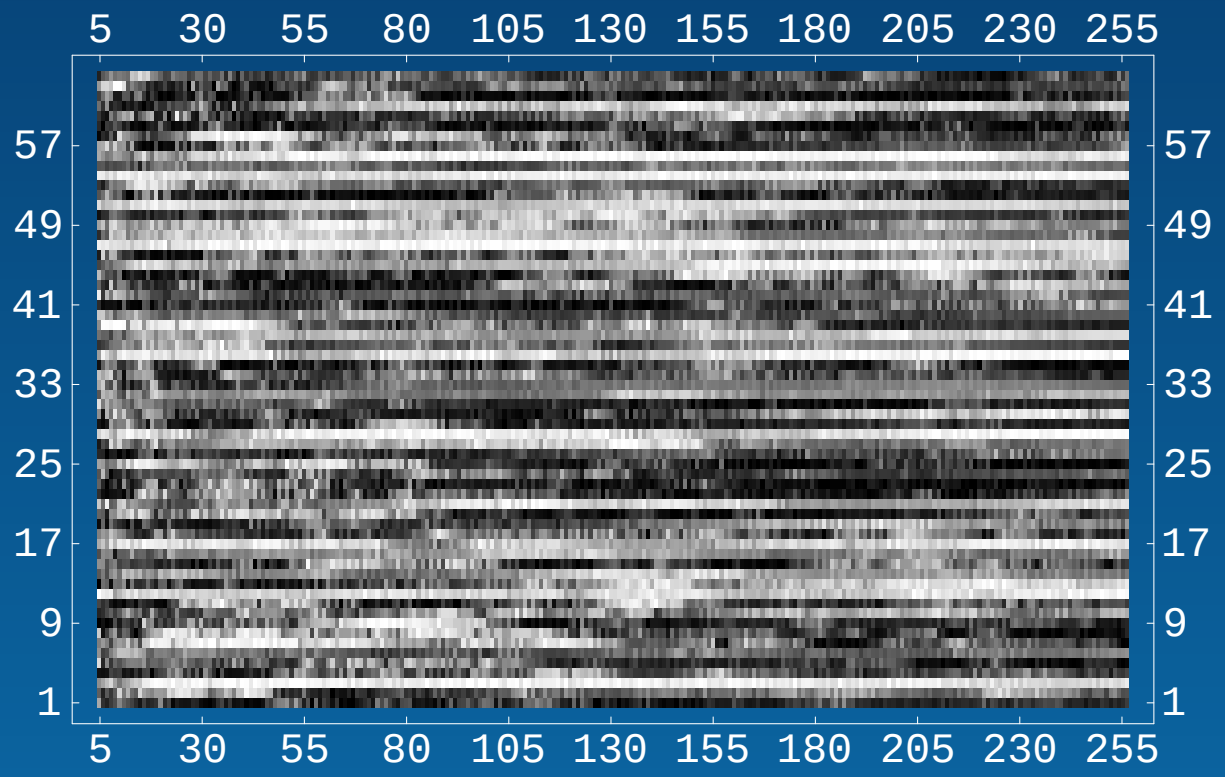
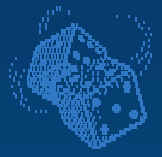


Fig. 6: Single Gambling Tests. Memory size $\lambda \in \{5, \ldots, 256\}$, 64 independent samples of upper-tail probabilities of the Gambling Test.

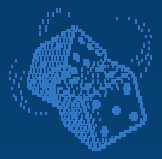


The End

Contact:

Stefan.Wegenkittl@sbg.ac.at
http://random.mat.sbg.ac.at/team/ste.html

Slides on web server.



Links and References

References

[1] S. Wegenkittl. Monkeys, gambling, and return times: Assessing pseudorandomness. In P.A. Farrington, H.B. Nembhard, D.T. Sturrock, and G.W. Evans, editors, *Proceedings of the 1999 Winter Simulation Conference*, pages 625–631. IEEE Press, 1999.

[2] S. Wegenkittl. Entropy estimators and serial tests for ergodic chains. *IEEE Transactions on Information Theory*, **47**(6):2480–2489, Sep 2001.

[3] S. Wegenkittl. Gambling tests for pseudorandom number generators. *Mathematics and Computers in Simulation*, 55(1–3):281–288, 2001.

[4] S. Wegenkittl. A generalized ϕ-divergence for asymptotically multivariate normal models, 2002. To appear in *Journal of Multivariate Analysis, Vol. 84, No. 1, 2003*, available via IDEALFirst.

[5] S. Wegenkittl and M. Matsumoto. Getting rid of correlations among pseudorandom numbers: Discarding versus tempering. *ACM Trans. Modeling and Computer Simulation*, **9**(3):282–294, 1999.

[6] S. Wegenkittl, "Generalized ϕ-Divergence and Frequency Analysis in Markov Chains,"

Ph.D. thesis, Universität Salzburg, Österreich, 1998, HTML version: http://random.mat.sbg.ac.at

[7] P. L'Ecuyer, R. Simard, and S. Wegenkittl, "Sparse serial tests of uniformity for random number generators," Accepted for publication in SISC, 2002.

[8] H. Leeb and S. Wegenkittl, "Inversive and linear congruential pseudorandom number generators in empirical tests.," *ACM Trans. Modeling and Computer Simulation*, vol. **7**, no. 2, pp. 272–286, 1997.

[9] S. Wegenkittl, "The pLab picturebook: Load tests and ultimate load tests, part I," Report no. 1, pLab – reports, University of Salzburg, 1997.

[10] P. Hellekalek, "Good random number generators are (not so) easy to find," *Mathematics and Computers in Simulation*, vol. **46**, pp. 485–505, 1998.

[11] U.M. Maurer, "A universal statistical test for random bit generators," *J. Cryptology*, vol. **5**, pp. 89–105, 1992.

[12] I. Csiszár, "Eine informationstheoretische Ungleichung und ihre Anwendung auf den Beweis der Ergodizität von Markoffschen Ketten," *Magyar Tud. Akad. Mat. Kutató Int. Közl*, vol. **8**, pp. 85–108, 1963.

[13] K. Pearson, "On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling.," *Philos. Magazine Series*, vol. **50** , no. 5, pp. 157–172, 1900.

[14] S. Kullback and R. Leibler, "On information and sufficiency," *Ann. Math. Statist.*, vol. 22, pp. 79–86, 1951.

[15] I. J. Good, "The serial test for sampling numbers and other tests for randomness," *Proc. Cambridge Philosophical Society*, vol. **49**, pp. 276–284, 1953.

[16] G. Marsaglia, "A current view of random number generators," in *Computer Science and Statistics: The Interface*, L. Billard, Ed. 1985, pp. 3–10, Elsevier Science Publishers B.V.

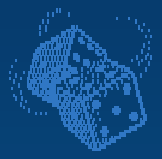
[17] G.H. Choe and D.H. Kim, "The probability distribution of the first return time," Submitted for publication, 1999.

[18] A. Wyner and J. Ziv, "Some asymptotic properties of the entropy of stationary ergodic data source with applications to data compression," *IEEE Trans. Information Theory*, vol. **35**, pp. 1250–1258, 1989.

[19] D. Ornstein and B. Weiss, "Entropy and data compression schemes," *IEEE Trans. Information Theory*, vol. **39**, pp. 78–93, 1993.

[20] J. S. Coron and D. Naccache, "An Accurate Evaluation of Maurer's Universal Test," in *Proceedings of Selected Areas in Cryptography 98*. 1998, Lecture Notes in Computer Science 1556, pp. 57–71, Springer.

[21] G.H. Choe and D.H. Kim, "Average convergence rate of the first return time," Submitted for publication, 1999.



Jumpstation

New Challenges for Empirical Testing
Entropy Primer
Computing Entropy
Tests for Randomness based on Entropy
Construction of Tests
Sample Study
Results of Study
Correlation between Tests
Application to Cryptographic Generator
Links and References